Distance Learning at the Cleveland Museum of Art

Tessellation Exploration!

Grades 7-12

This packet includes:

Teacher notes:

Please have students bring pencil and paper to the distance learning program, along with the Viewing Guide. Students will be completing this activity sheet during the lesson.

*Optional - If you would like the class to do the "Everybody Tessellate" activity during the conference, please bring the suggested materials. (Photocopies of ***Everybody Tessellate!*****,** 8.5" x 11" card stock or paper, pencils, markers, rulers, scissors, index cards, scotch tape.)

Tessellation Exploration!

Grades 7-12

Teacher Information Guide:

Program Objectives:

Students will learn and understand…

1. How different cultures have used math in the form of tessellations in their artwork.
2. The difference between various types of tessellations.
3. How math can be used to create art.

National Education Standards:

For Fine Arts - Visual Arts (grades 5-8, 9-12):

- Understanding and applying media, techniques, and processes.
- Using knowledge of structures and functions.
- Understanding the visual arts in relation to history and cultures.
- Making connections between visual arts and other disciplines.

For Mathematics – Geometry (grades 6-8, 9-12):

- Analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
- Apply transformations and use symmetry to analyze mathematical situations.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

For Language Arts - English (grades K-12):

- Evaluation Strategies
- Communication Skills
- Applying Knowledge
- Evaluating Data
- Developing Research Skills

Program Description:

The Cleveland Museum of Art has an extensive collection of tessellated art. It is with some of these pieces that this lesson was written. The use of math and tessellations in art is more common than most people think. Different cultures have used patterns and tessellations in their artwork as a decorative and symbolic part of their heritage. Artists have been mesmerized by the beautiful complexity of tessellations, trying to imitate the forms that they see in architecture and nature.

In this lesson, we will explore how math is used to create a variety of objects, from intriguing artwork of the ancient Egyptians, to the intricate work of M.C. Escher. The museum's collection will be used to illustrate mathematical concepts about polygons and different kinds of tessellations. Students will see that math is used extensively in the creative world of art and architecture.

Prior to the Program:

1. Have students review the vocabulary. They should have a clear understanding of the terms provided.
2. Discuss how art and math can be related. What artists and cultures have used math and/or tessellations in their artwork?

Selected Vocabulary:

Polygon - a figure formed by three or more segments connecting only at their end points

Vertex of a polygon - point where two sides meet. Two or more of these points are called vertices.

Side - one of the line segments making up the figure

Convex Polygon - a polygon in which no interior angle is greater than 180 degrees (all of the interior angles "point out.") A line segment drawn between any two vertices of a convex polygon would remain inside the boundaries of the figure.

Concave polygon - a polygon is concave if at least one of its internal angles is greater than 180 degrees. A concave polygon must have at least four sides. (It appears to "cave in.") In a concave polygon, you could draw a line segment between a pair of vertices that would fall outside of the boundaries of the figure.

Regular Polygon - a polygon that is equilateral and equiangular

Tessellation - an arrangement of closed figures, such that the plane is completely covered with no gaps between figures

Regular tessellation - a tessellation, or tiling, involving one regular polygon (only possible with squares, equilateral triangles and hexagons)

Non-regular tessellation - a tessellation using non-regular polygons

Schläfli symbol - this is the notation used to specify tessellations, such as **{6, 3}**. The first number (6) is the number of side the polygon has, and the second number (3) is number of polygons that touch at each vertex of the tessellation.

Pure tessellation - a tessellation using only one shape (could be regular or non-regular)

Semi-pure tessellation - a tessellation involving more than one shape

Semiregular (Semi-regular) tessellation - a semi-pure tessellation of regular polygons (more than one regular polygon is used) in which the arrangement at each vertex is the same. (The same shapes appear, in the same sequence.)

Demiregular (Demi-regular) tessellation - a semi-pure tessellation of regular polygons (more than one regular polygon is used) in which the arrangement at each vertex is **not** the same

Teaching Extensions:

1. Language Arts

Have students choose and research an artist or culture that uses tessellations. Allow students to present their findings to the class through a PowerPoint presentation.

Materials needed: Computers, internet, PowerPoint program, research books and materials

2. Math

Photocopy the enclosed Math worksheets, Tessellations: The Angle Sum of a Convex Polygon, for each student to complete during the lesson.

3. Visual Arts

(This activity can be done during the videoconference, if the class brings the materials to the session.)

Photocopy ***Everybody Tessellate!*** for each student. Have students make their own tessellation in the form of M.C. Escher.

Materials needed: Photocopies of ***Everybody Tessellate!*****,** 8.5" x 11" card stock or paper, pencils, markers, rulers, scissors, index cards, scotch tape.

TEACHER RESOURCE LIST

Recommended Reading:

Locher, J.L., *M.C. Escher: Life and Work.* New York: Harry N. Abrams, 1992.

Stephans Geiger, Pamela. *Tessellations: History and Making of Symmetrical Designs.* Illinois: Crystal productions, 2001.

Willson, John. *Mosaic and Tessellated Patterns: How to Create Them*. New York: Dover Productions, 1983

Web Sites:

For students...

Tessellations.org **http://www.tessellations.org/index.htm**
Provides samples, descriptions and do it yourself instructions.

Tessellate! **http://www.shodor.org/interactivate/activities/tessellate/**
A short activity that allows students to see how certain shapes tessellate.

Math is Fun **http://www.mathisfun.com/geometry/tessellation-artist**

An activity to create your own tessellation.

For teachers…

Platonic Realms. The Mathematical Art of M.C. Escher.
http://www.mathacademy.com/pr/minitext/escher/index.asp
An introduction to Escher's work and how he incorporated math into his art.

MathWorld. **http://mathworld.wolfram.com/Tessellation.html**
Provides great descriptions of tessellations and the geometry involved in tessellations.

This Teacher Information Packet and Distance Learning lesson were developed with assistance of Kristin LaGuardia, Teacher, Cuyahoga Community College, Cleveland, Ohio and Lenaia Burbank, Educator, The Cleveland Museum of Art.

Name:_____

Tessellation Exploration! Viewing Guide

Below are three pieces of art that are shown in the lesson that are made of a regular tessellation. What type of polygon(s) is used in each piece?

A. *Mummy Case* -

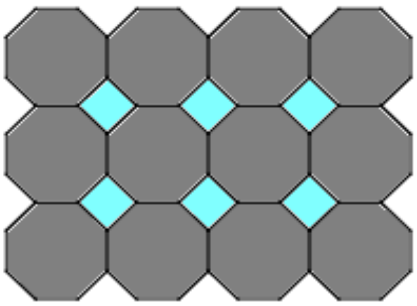
B. *A Bishop Saint with Donor* -

C. *The Terrible Adventures of Scholastica* -

Define and draw an example of each of the following tessellations:

Types of Tessellation	Definition	Diagram/Example
Regular Tessellation		
Semi-Pure Tessellation		
Demi-Regular Tessellation		
Non-Regular Tessellation		

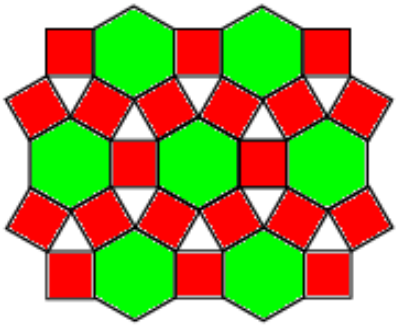
Tessellation Examples:



Semiregular Tessellation Example 1

What polygons do you see at each vertex?

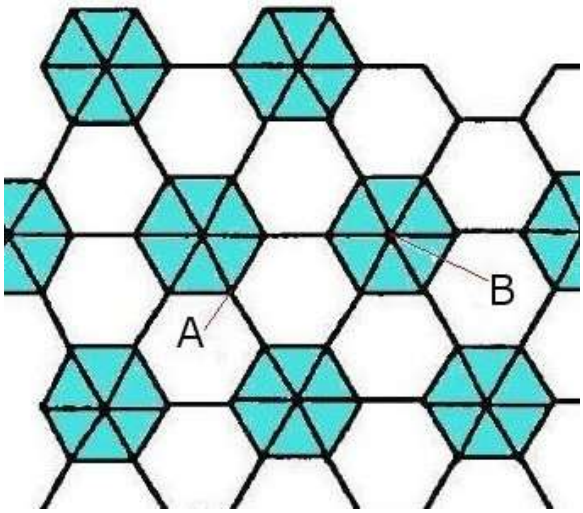
_____
_____
_____



Semiregular Tessellation Example 2

What polygons do you see at each vertex?

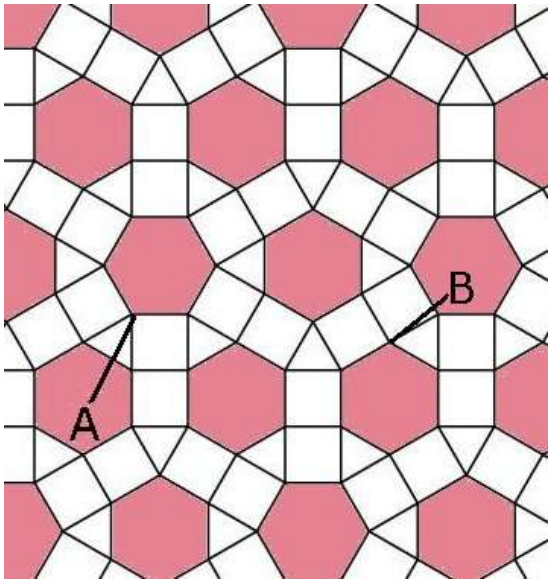
_____
_____
_____



At vertex A:

At vertex B:

Demiregular Tessellation Example 1



At vertex A:

At vertex B:

Demiregular Tessellation Example 2

Tessellation Exploration! Viewing Guide – Answers

Below are three pieces of art that are shown in the lesson that are made of a regular tessellation. What type of polygon(s) is used in each piece?

A. *Mummy Case* - **Squares**

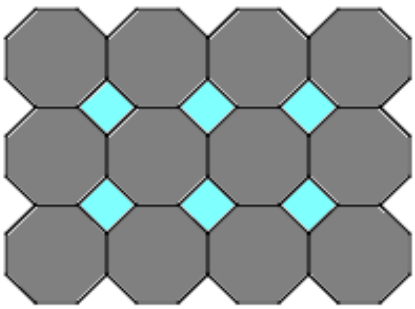
B. *A Bishop Saint with Donor* - **Squares**

C. *The Terrible Adventures of Scholastica* – **Hexagons (on floor)**

Define and draw an example of each of the following tessellations:

Types of Tessellation	Definition	Diagram/Example
Regular Tessellation	**It is one involving regular polygons. Regular polygons are polygons that are equilateral and equiangular**	
Semi-Pure Tessellation	**A tessellation involving more than one shape**	
Demi-Regular Tessellation	**A semi-pure tessellation of regular polygons in which the arrangement at each vertex is not the same**	
Non-Regular Tessellation	**A tessellation using non-regular polygons.**	

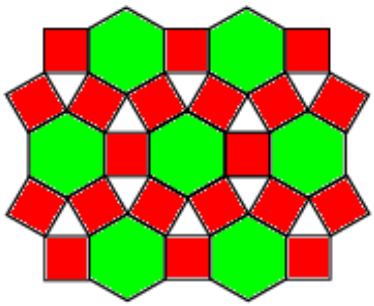
Tessellation Examples:



Semiregular Tessellation Example 1

What polygons do you see at each vertex?

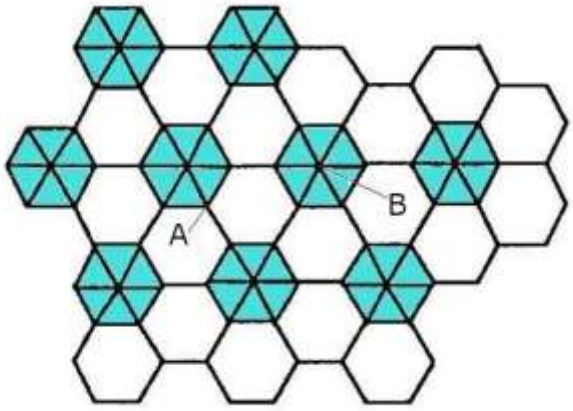
Octagon
Octagon
Square



Semiregular Tessellation Example 2

What polygons do you see at each vertex?

Hexagon
Square, Triangle,
Square



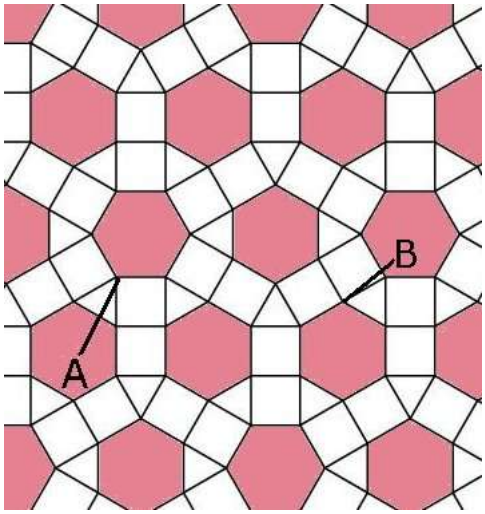
At vertex A:

Hexagon
Hexagon
Triangle
Triangle

At vertex B:

Triangle, Triangle
Triangle, Triangle
Triangle,
Triangle

Demiregular Tessellation Example 1



At vertex A:

Hexagon
Square
Triangle
Square

At vertex B:

Hexagon
Square
Square
Triangle

Demiregular Tessellation Example 2

Tessellation Exploration!: The Angle Sum of a Convex Polygon
Part I

Using a straightedge, draw a triangle.

What is the angle sum of a triangle? _____

Using a straightedge, draw any convex quadrilateral ABCD. Draw diagonal AC.

What is the angle sum of triangle ABC?_____
What is the angle sum of triangle ACD?_____

What conclusion can you draw about the angle sum of quadrilateral ABCD? _____

Tessellations:
Part II

Using a straightedge, draw any convex pentagon ABCDE. From vertex A, draw all of the possible diagonals.

Pentagon ABCDE is now divided into _____ triangles.
What is the angle sum of ABCDE? _____

Continue this pattern with the next polygon. Draw a convex hexagon and draw all of the diagonals from one of the vertices. Look for a pattern relating the number of sides of the polygon to the number of triangles into which the polygon is divided. Once you have determined the number of triangles, multiply that number by the triangle angle sum to determine the angle sum of the polygon itself.

What is the angle sum of the hexagon?_____

Tessellations:
Part III

Now let's generalize this for a polygon with *n* sides.

A polygon with *n* sides can be divided into _____ triangles by drawing all of the possible diagonals from one vertex.

The angle sum formula for a polygon with *n* sides is _____.

Use the formula you derived to determined the angle sum of the following polygons:

a. Heptagon _____

b. Octagon _____

c. Nonagon _____

d. Decagon _____

e. 15-gon _____

f. 47-gon _____

Tessellation Exploration!: The Angle Sum of A Convex Polygon - ANSWER KEY
Part I

Using a straightedge, draw a triangle.

What is the angle sum of a triangle? **180**

Using a straightedge, draw any convex quadrilateral ABCD. Draw diagonal AC.

What is the angle sum of triangle ABC? **180**
What is the angle sum of triangle ACD? **180**

What conclusion can you draw about the angle sum of quadrilateral ABCD? **360 Degrees, the sum of two triangles**

Tessellations:
Part II

Using a straightedge, draw any convex pentagon ABCDE. From vertex A, draw all of the possible diagonals.

Pentagon ABCDE is now divided into **3** triangles.
What is the angle sum of ABCDE? **3(180) = 540**

Continue this pattern with the next polygon. Draw a convex hexagon and draw all of the diagonals from one of the vertices. Look for a pattern relating the number of sides of the polygon to the number of triangles into which the polygon is divided. Once you have determined the number of triangles, multiply that number by the triangle angle sum to determine the angle sum of the polygon itself.

What is the angle sum of the hexagon? **720**

Tessellations:
Part III

Now let's generalize this for a polygon with *n* sides.

A polygon with *n* sides can be divided into **n-2** triangles by drawing all of the possible diagonals from one vertex.

The angle sum formula for a polygon with *n* sides is **(n-2)180**.

Use the formula you derived to determined the angle sum of the following polygons:

g. Heptagon **900**

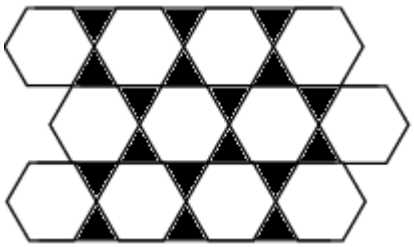
h. Octagon **1080**

i. Nonagon **1260**

j. Decagon **1440**

k. 15-gon **2340**

l. 47-gon **8100**

Everybody  **Tessellate!**

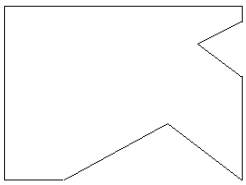
Everyone has a little M.C. Escher inside of them. Follow the steps below to create your own tessellation.

- Using the index card cut it into a square or rectangle.
- Cut shapes out of the square. (Step 2) You can cut any shape you would like.
- Take the cut out shape and tape it to the opposite side of the square. (Step 3)
- You now have your template!
- Place your template on the 8.5" x 11" card stock or paper and trace it.
- Move the template over and trace it again so that it interlocks with the other image.
- Continue this until the entire sheet has been filled.

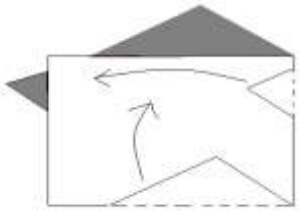
Some things to think about when planning your tessellation:

- How complex do you want to make your figure?
- How will you use color or imagery to alter your tessellation?
- Do you have a theme?

Step 1



Step 2



Step 3

The Cleveland Museum of Art
Selected Images for Tessellation Exploration!

Casement Window
Frank Lloyd Wright, 1904, Glass.
1984.11

Crown (ade), Nigeria, Yoruba people,
1900's, cloth, glass beads, wood, feather quills. 1995.22

The Cleveland Museum of Art
Selected Images for Tessellation Exploration!

Floor Mosaic Panel: Grape Harvester with Peacock, c. 400's, Marble Tesserae.
1969.112

Vessel, Lucy Martin Lewis, 1900's,
Ceramic and Slip. 2003.345